

EVENT TREE FORMULATION OF SYSTEM RELIABILITY

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1. Introduction

Buehler's "initial" bounds for the reliability function of a series system and the bounds for the reliability function of monotone systems exploit the fact that in both cases, the maximum of the reliability function over a cartesian product region is achieved at the upper bounds of the individual arguments (Guerrero and David, 1985); i.e.,

$$\max \{H(p_1, p_2, \dots, p_n)\} : p_i \leq \bar{p}_i;$$

$$i=1, 2, \dots, n = H(\bar{p}_1, \bar{p}_2, \dots, \bar{p}_n).$$

Pavlov (1980) provides sufficient conditions under which the same type of computation may be made for functions H that are quasi-convex whose arguments are parameters of logarithmically convex distributions.

In this paper, a class of functions is presented for which it is generally less clear how a reasonable set of initial bounds that provide an ordering of the sample space is to be constructed. We then establish an algorithm that shows that, for a certain subclass of such functions maximization over a cartesian product region is attained at a vertex point of the region; i.e., the functions maximized at lower and upper bounds of the arguments of the function. In this connection, it will be useful to work with "event tree" representations of systems and the notion of an "F-event tree". This property is exploited to obtain a confidence

bound procedure for the probability of F-tree events that is optimal for a particular class of confidence procedures. If Buehler bounds for the function are desired, the bounds provided by this optimal procedure would be our recommended initial bounds for ordering the sample space.

2. Event Trees

A time-inclusive representation of a system in terms of the elementary events that contribute to system success is the event tree (Wong, 1984). Alternative representations are provided by the interrelated constructs of fault trees and circuit diagrams (Barlow and Lambert, 1975). At any rate, the tree provides a useful structure for the analysis of the probability of occurrence of a composite event of interest, through its inclusion of all events that contribute to this composite event.

Event trees are conveniently portrayed in terms of the nodes, arcs and branches of a finite directed topological tree, as defined in graph theory (Even, 1973). In the context of this discussion, direction is provided by the elapsing of time, each node represents a component of the system (more properly: the utilization of a component), each arc designates a component state and each branch (i.e., a directed path from the "initial" node of the tree to a "terminal" node of the tree) traces out a basic event. It will be useful to label an arc with the (conditional) probability of the component

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utilization outcome it represents; the probability of a basic event, say $p(E)$, will then be the product of the (conditional) probabilities along the corresponding branch.

An example of an event tree and some of the terminology associated with it is given in Figure 1. Figure 1 also implicitly defines the notions of "initial" node and "terminal" node and the "level" of a node. These terms are formally defined below.

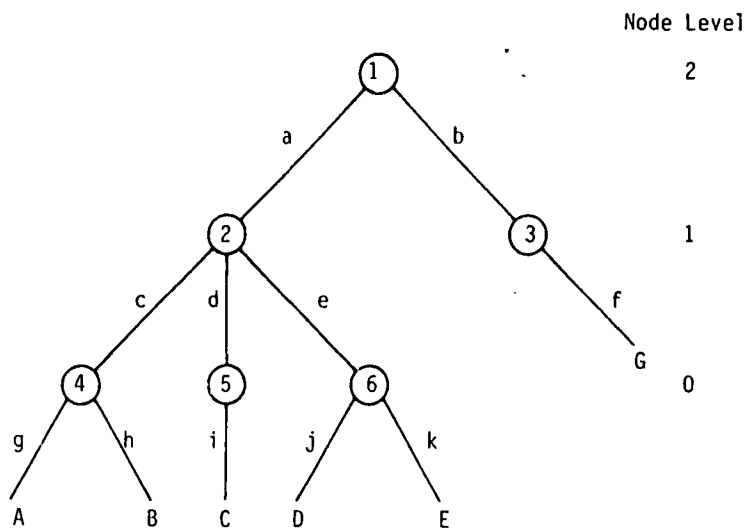


Figure 2.1. An event tree with six components

Notes:

1. Node 1 is the initial node.
2. Nodes 3, 4, 5, and 6 are terminal nodes.
3. Node levels are numbered starting with 0 for the last level.
4. The probability of basic event B is the product of ach.

Definition 2.1: Node A is called the initial node of an event tree if there exists a (directed) path from node A to every node of the tree.

Definition 2.2: Node A is a terminal node of an event tree if there is no (directed) path from node A to any other node of the tree.

Definition 2.3: The length of the path from node A to node B is the number of arcs in the path.

This definition implies that the length of a branch is the number of arcs from the initial node of the tree to the terminal node of the branch.

Definition 2.4: The level of node A is given by $N-i$, where N is the maximum length of the branches of the tree and i is the length of the path from the initial node of the tree to node A.

Thus, the level of the initial node is N , and there is at least one terminal node of level 0.

Suppose we distinguish between only two states: a component either functions or fails. The event tree, without the arc labels, for such a system is sometimes referred to as a binary tree (Horowitz and Sahni, 1978). An example of a binary tree with seven nodes is shown in Figure 2.

For binary event trees, let us denote the (conditional) component utilization outcome probabilities by:

p_k = probability that the k th component utilization is successful, given the history of component utilizations, successes and failures corresponding to the sequence of arcs leading from the initial node to node k

and

$$q_k = 1 - p_k, \quad k=1, 2, \dots, n,$$

where n is the number of component utilizations (i.e., the number of nodes) of the event tree.

Since the basic events are disjoint, the probability of a composite event is simply the sum of the probabilities of the basic events of which it is the union. Let $p = (p_1, p_2, \dots, p_n)$ be the vector of (conditional) probabilities defined above and let $H(p)$ denote the probability of a composite event. Let us now consider the following maximization problem:

$$\max \{H(p) : p \in Q\} = \max_Q H(p) \quad (2.1)$$

where $Q = \prod_{k=1}^n Q_k$, $Q_k = \{p_k : \underline{p}_k \leq p_k \leq \bar{p}_k\}$ and

\underline{p}_k and \bar{p}_k are lower and upper bounds, respectively, for p_k , $k = 1, 2, \dots, n$.

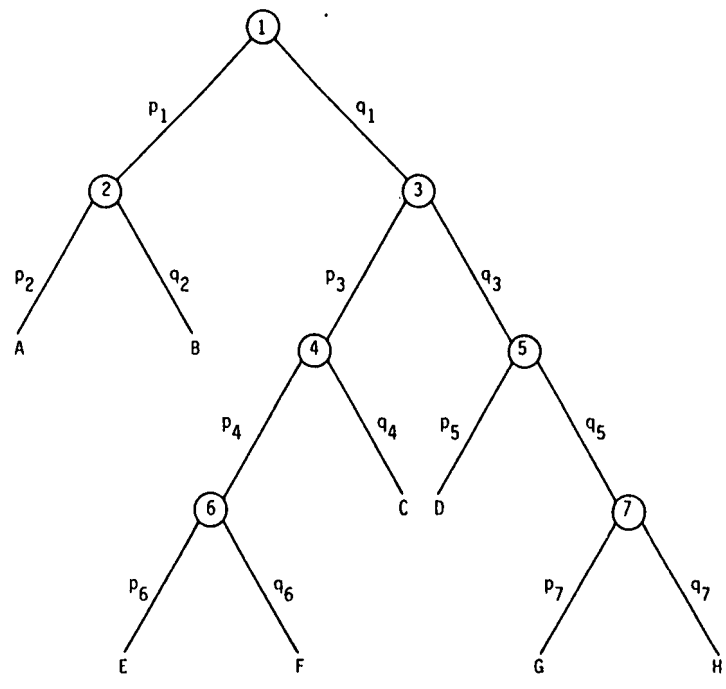


Figure 2. An F-event tree illustration

In general, it will not be true that the optimal value of $H(p)$ is attained at a point, say p^o , for which p_k^o is either \underline{p}_k or \bar{p}_k for all k . In the next section, we present a sub-class of functions $H(\cdot)$ suggested by S. Fahrenholtz, for which it is always true that the solution to the maximization problem stated above in Equation 2.1 is attained at a point P^o whose arguments are either the upper or lower bounds for each p_k as specified in the set Q_k .

3. Bounding the Probability of an F-event

3.1 Definition and Notations

Consider a system whose event tree representation is a binary tree. The event tree for such a system will be referred to as an F-event tree if each (conditional) utilization outcome probability appears exactly once. By this definition, the event tree exhibited in Figure 2 is an F-event tree; on the other hand, the event tree exhibited in Figure 3 is not an F-event tree. Any union of basic events in an F-event tree will be referred to as an F-event.

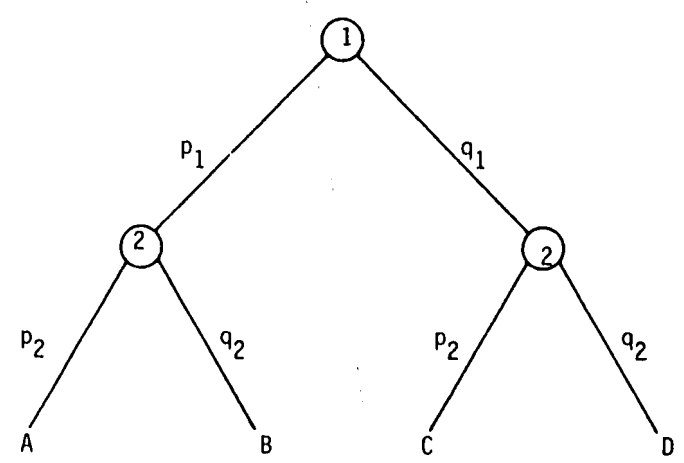


Figure 3. An event tree that is not an F-event tree

Since the probability of a basic event is the product of (conditional) utilization outcome probabilities on its associated branch, and since every branch contains only one node from each level, each factor in this product is associated with a different level, with each level between the initial node and the terminal node of the branch, inclusive, contributing a factor to the product. In addition, every factor that appears in the product appears only once, and the probability of an F-event is multilinear since it is the sum of such products.

Let $H(p)$ denote the probability of an F-event where p is the vector of (conditional) utilization outcomes of the components of the F-event tree. The rest of this section will be devoted to establishing an algorithmic solution to the maximization problem defined in Equation 2.1 when the objective function H is the probability of an F-event.

To illustrate what is entailed in solving this maximization problem, the following example is given.

Example 1 : Let $H(p) = p_1 p_2 q_4 + q_1 p_3 p_6$. Since Q is the cartesian product of the intervals Q_k , the values assumed by each p_k in the region Q do not depend on the values assumed by the other arguments. This is in contrast to the situation where, for example, $Q = \{(p_1, p_2): p_1 + p_2 \leq 1\}$ in which case the possible values of p_1 depend on the values of p_2 and vice-versa. One implication of the lack of dependence of the Q_k 's is that the maximization of $H(p)$ over Q is equivalent to the maximization of $H(p)$ over $Q' = Q_1 \times Q_2 \times Q_3 \times Q_4 \times Q_6$. Next, note that the maximizing values of q_4 and p_6 , both of

which are associated with terminal nodes, do not depend on the values of the factors that precede them; i.e., their maximizing values do not depend on values of the state probabilities of the nodes that are on levels 2 and 3. Consider now the iterative evaluation:

$$\max_{Q'} H(p) = \max_{Q' - Q^{(0)}} \max_{Q^{(0)}} H(p) \quad (3.1)$$

where $Q^{(0)} = Q_4 \times Q_6$. In view of the above remark concerning p_6 and q_4 , and since q_4 and p_6 appear in separate terms, we may write, for all (p_1, p_2, p_3) in $Q_1 \times Q_2 \times Q_3$,

$$\begin{aligned} \max_{Q^{(0)}} H(p) &= \max_{Q_4} \max_{Q_6} H(p) \\ &= \max_{Q_6} \max_{Q_4} H(p) \\ &= p_1 p_2 (1 - p_4) + q_1 p_3 \bar{p}_6 \end{aligned} \quad (3.2)$$

Substituting Equation 3.1 into Equation 3.2, we may write, with $Q^{(1)} = Q_2 \times Q_3$:

$$\begin{aligned} \max_{Q'} H(p) &= \max_{Q' - Q_1} p_1 p_2 (1 - p_4) + q_1 p_3 p_6 \\ &= \max_{Q_1} \max_{Q^{(1)}} p_1 p_2 (1 - p_4) + q_1 p_3 p_6 \end{aligned}$$

Again, since p_2 and p_3 both belong to level 1 and hence appear in separate terms, we have:

$$\begin{aligned} \max_{Q^{(0)}} p_1 p_2 (1 - p_4) + q_1 p_3 \bar{p}_6 \\ &= \max_{Q_2} \max_{Q_3} p_1 p_2 (1 - p_4) + q_1 p_3 p_6 \\ &= \max_{Q_3} \max_{Q_2} p_1 p_2 (1 - p_4) + q_1 p_3 \bar{p}_6 \\ &= p_1 p_2 (1 - p_4) + q_1 \bar{p}_3 \bar{p}_6. \end{aligned}$$

Finally, note that

$$\begin{aligned} p_1 p_2 (1 - p_4) + q_1 p_3 p_6 \\ = p_1 p_2 (1 - p_4) - p_3 p_6 + p_3 p_6 \end{aligned}$$

which implies that the value of p_1 , say p_1 , that maximizes $H(p)$ is given by:

$$p_1 = \begin{cases} p_1 & \text{if } \bar{p}_2(1-p_4) \leq \bar{p}_3\bar{p}_6 \\ \underline{p}_1 & \text{otherwise} \end{cases}$$

Thus H is maximized at interval endpoints and the proper choice of endpoint is iteratively determined. These features are systematically exploited in the algorithm discussed in the remainder of this section.

The following notations will be helpful: for a specified event E of an F -event tree, let

$\pi(k)$ = product of (conditional) component utilization outcome probabilities from the initial node to node k ;

$\pi(s_k)$ = product of (conditional) component utilization outcome probabilities of nodes that follow the functioning state of node k ;

$\pi(f_k)$ = product of (conditional) component utilization outcome probabilities of nodes that follow the failed state of node k ;

$E(k)$ = probability of all basic events in event E that involve node k ;

$E'(k)$ = probability of all basic events in event E that do not involve node k . (3.3)

3.2 An algorithm for bounding the probability of an F -Event

Using the above notations, the probability of an F -event can be represented by the decomposition:

$$H(p) = E(k) + E'(k) \quad (3.4)$$

and $E(k)$ can be expressed as:

$$\begin{aligned} & \pi(k) p_k \pi(s_k) + q_k \pi(f_k) \\ & \text{if both } p_k \text{ and } q_k \text{ appear in } H \\ & \pi(k) p_k \pi(s_k) \text{ if only } p_k \text{ appears in } H \end{aligned}$$

$$\pi(k) q_k \pi(f_k) \text{ if only } q_k \text{ appears in } H. \quad (3.5)$$

Expressions 3.4 and 3.5 clearly establish the following:

Property 3.1: p_k and q_k appear only in $E(k)$ and both appear as shown in expression 3.6.

Property 3.2: None of the arguments in $\pi(s_k)$ and $\pi(f_k)$ appear in $E'(k)$ or $\pi(k)$ and any p_j or q_j is a factor in only one of $\pi(k)$, $\pi(s_k)$ or $\pi(f_k)$.

Property 3.3: The arguments p_j corresponding to nodes that are on the same level of the event tree as node k appear only in $E'(k)$.

In what follows, we will assume that the following condition is met:

Condition 1: If k is a terminal node, then only one of its states occurs in an event. This is a reasonable condition to impose because if both states of a terminal node occur, then expressions 3.4 and 3.5 imply:

$$\begin{aligned} H(p) &= E'(k) + \pi(k) p_k + \pi(k) q_k \\ &= E'(k) + \pi(k), \end{aligned}$$

which indicates that node k is not relevant to event E since both p_k and q_k do not appear in $H(p)$.

We also note that due to the cartesian product nature of Q , the maximization of $H(p)$ over Q is equivalent to the maximization of $H(p)$ over the cartesian product Q' of Q_k 's that correspond to the p_k 's that actually appear in $H(p)$.

We next establish a series of theorems and corollaries that provide the algorithmic solution to the problem of maximizing the probability of an F -event over a cartesian product Q' .

Theorem 1.: Let k be a terminal node of an F-event tree. Then,

$$\max_{Q'} H(p) = \max_{Q' - Q_k} E'(k) + \pi(k)p_k^*$$

where

$$p_k^* = \begin{cases} p_k & \text{if } p_k \text{ appears in } H \\ 1-p_k & \text{if } q_k \text{ appears in } H. \end{cases} \quad (3.6)$$

Proof: Condition 1 implies that $E(k)$ is either $\pi(k)p_k$ or $\pi(k)q_k$. In either case, by Property 3.1 and the cartesian product nature of Q' , the value of p_k that maximizes H over Q' is the value of p_k that maximizes H over Q_k which is exactly the quantity defined in expression 3.6 above. Q.E.D.

The following corollary is implied by Theorem 1. and Property 3.3 and establishes that the maximizing values of probabilities associated with terminal nodes are obtained independently of each other and of non terminal nodes and are attained at the endpoints of the intervals Q_k .

Corollary 1.: Let $Q^{(0)}$ be the cartesian product of the Q_k 's corresponding to terminal nodes of an F-event tree. Let P_A be the vector of all p_i 's corresponding to non-terminal nodes appearing in event E and let P_B be the vector of all p_i 's corresponding to the terminal nodes appearing in event E . Then,

$$\begin{aligned} \max_{Q'} H(p) &= \max_{Q' - Q^{(0)}} \max_{Q^{(0)}} H(p_A, p_B) \\ &= \max_{Q' - Q^{(0)}} H(p_A, p_B^{\circ}) \end{aligned}$$

where p_B° is the vector of maximizing values defined in expression 3.6.

In what follows, the following decomposition of the vector of probabilities p

will be helpful:

$$p = (p_k, p_A, p_B)$$

where p_A is the vector of probabilities that appear in either $\pi(k)$ or $E'(k)$ and p_B is the vector of probabilities that appear in either $\pi(s_k)$ or $\pi(f_k)$. We next show that the value of p_k that maximizes H are obtained independently of the values of the vector p_A and depends on the vector p_B only through its maximizing value.

Theorem 2.: Let k be a nonterminal node.

Then,

$$\begin{aligned} \max H(p) &= \max_{Q_A} \max_{Q_k} \max_{Q_B} H(p_k, p_A, p_B) \\ &= \max_{Q_A} \max_{A_k} H(p_k, p_A, p_B^{\circ}) \end{aligned}$$

where p_B° is the value of p_B that maximizes H .

Proof: We consider three cases.

Case 1. Suppose only p_k appears in H .

Then,

$$H(p) = E'(k) + \pi(k)p_k \pi(s_k).$$

By property 3.2, the maximizing value of the linear product $\pi(s_k)$ is obtained independently of the values of the p_i 's in $E'(k)$ and $\pi(k)$. Now $\pi(s_k)$ is the product of factors p_i or $1-p_i$ for which p_i belong to p_B ; hence, the maximizing value of $\pi(s_k)$ is attained at some p_B° ; i.e.:

$$\begin{aligned} \max_{Q'} H(p) &= \max_{Q_A} \max_{Q_k} \max_{Q_B} E'(k) + \pi(k)p_k \pi(s_k) \\ &= \max_{Q_A} \max_{Q_k} E'(k) + \pi(k)p_k \pi^{\circ}(s_k) \end{aligned} \quad (3.7)$$

Case 2. Suppose only q_k appears in H .

Then, using the same arguments as above, we may write:

$$\begin{aligned} \max H(p) &= \max_{Q_A} \max_{Q_K} \max_{Q_B} E'(k) + \pi(k)q_k \pi(f_k) \\ &= \max_{Q_A} \max_{Q_K} E'(k) + \pi(k)q_k \pi^{\circ}(f_k). \end{aligned} \quad (3.8)$$

Case 3. Suppose both p_k and q_k and q_k appear in H. then,

$$\begin{aligned} H(p) &= E'(k) + \pi(k) \{ p_k \pi(s_k) + q_k \pi(f_k) \} \\ &= E'(k) + \pi(k) \{ p_k \{ \pi(s_k) - \pi(f_k) \} + \pi(f_k) \}, \end{aligned}$$

Property 2.3 implies that the values of $\pi(s_k)$ and $\pi(f_k)$ that maximize H are obtained independently of the values of p_k and p_A and are attained at some p_B° ; hence, we may write,

$$\begin{aligned} \max_{Q'} H(p) &= \max_{Q_A} \max_{Q_K} \max_{Q_B} E'(k) + \pi(k) \\ &\quad + \pi(k) \{ p_k \{ \pi(s_k) - \pi(f_k) \} + \pi(f_k) \} \\ &= \max_{Q_A} \max_{Q_K} E'(k) + \pi(k) \{ p_k \{ \pi^{\circ}(s_k) \\ &\quad - \pi^{\circ}(f_k) \} + \pi^{\circ}(f_k) \} \end{aligned} \quad (3.9)$$

Q.E.D.

Corollary 2.: The maximum value of H over the cartesian product Q' is attained at a point $p^{\circ} = (p_1^{\circ}, p_2^{\circ}, \dots, p_n^{\circ})$ for which p_k° is either p_k or p_k .

Proof: If k is a terminal node, then Theorem 1 establishes the result. Suppose k is a nonterminal node. Then, Equations 3.7 through 3.9 in the proof of Theorem 3.6 above imply that:

- i) if only p_k appears in H, then $p_k^{\circ} = p_k$,
- ii) if only q_k appears in H, then $p_k^{\circ} = p_k$,
- iii) if both p_k and q_k appear in H then,

$$p_k^{\circ} = \begin{cases} p_k & \text{if } \pi^{\circ}(s_k) \geq \pi^{\circ}(f_k) \\ p_k & \text{otherwise} \end{cases}$$

Q.E.D.

We can summarize the algorithmic procedure as follows:

(1) Begin by obtaining maximizing values of p_i 's corresponding to terminal nodes.

(2) Find the maximizing values of all p_i 's corresponding to nodes on level r before proceeding to p_i 's corresponding to nodes on level r+1.

(3) For each p_i , the maximizing value is determined by (i), (ii) and (iii) in the proof of Corollary 2 above.

4. Some Generalizations Concerning Event Tree Probability Functions

4.1 Characterizing the probability of an F-event

The essential features of the probability H(p) of an F-event may be summarized as follows:

(1) H is defined on a cartesian product

$$\pi_{k=1}^{Q_k} = \pi_{k=1} [p_k, \bar{p}_k].$$

(2) H is multilinear in its arguments p_k .

(3) It is possible to arrange the arguments in a sequence, say $p^{(1)}, p^{(2)}, \dots, p^{(k)}, \dots, p^{(n)}$, such that the iterative maximization of this multilinear function over Q creates a conditional maximization for the argument $p^{(k)}$ that can be carried out independently of the "prior" arguments $p^{(1)}$ through $p^{(k-1)}$ and that depends on the arguments $p^{(k+1)}$ through $p^{(n)}$ only through their maximizing values.

(4) H attains its maximum at a vertex of Q

One may give up (2) and still have the important property (3), except that (4) will no longer necessarily obtain. Also, it is equally true that one can lose the critical property (3) yet keep (2) and (4). The reliability function of a k-out-of-n system is an example of such a function.

A function that satisfies the properties enumerated in (1) through (4) above will be

referred to as an F-function. If H is an F-function, property 4 implies that the maximization problem:

$$\max \{H(p): p_k \leq p_k \leq \bar{p}_k, k = 1, 2, \dots, n\}$$

is equivalent to the discrete maximization problem

$$\max H(p): p_k \in \{p_k, \bar{p}_k\}, k = 1, 2, \dots, n.$$

In other words, one may view the region of optimization as the set of 2^n points (p_1, p_2, \dots, p_n) for which p_i is either p_i or \bar{p}_i for $i = 1, 2, \dots, n$. Each one of these 2^n points may be associated with a cartesian product region with sides:

$$\begin{aligned} p_i \leq p_i \leq 1 & \quad \text{if } p_i = p_i \\ 0 \leq p_i \leq p_i & \quad \text{if } p_i = \bar{p}_i. \end{aligned} \quad (4.1)$$

Hence, this maximization problem may also be interpreted as finding the region belonging to the set of 2^n cartesian product regions, say $P(k)$, whose sides are either $[p_i, 1]$ or $[0, p_i]$ that minimizes:

$$\max \{H(p): p \in p(k)\} \quad \text{for } k=1, 2, \dots, 2^n.$$

To verify that this interpretation is valid note that if H attains its maximum value over Q at p_i for the argument p_i and $p(k)$ includes the side $[0, p_i]$ instead of $[p_i, 1]$, then the maximum of H over $p(k)$ will be larger than the optimal solution. Similarly, if H attains its maximum value over Q at p_i for the argument \bar{p}_i and $p(k)$ includes the side $[p_i, 1]$ instead of $[0, p_i]$, then the maximum of H over $p(k)$ will be larger than the optimal solution.

4.2 Restricted maximization for event tree probability functions

Suppose that the utilization outcome probabilities of an event tree are not all

distinct; i.e., the probabilities associated with two or more nodes of the event tree are equal. If the event E involves nodes that have the same utilization outcome probabilities then, the problem of bounding H(p) over a cartesian product Q may be expressed as:

$$\text{maximize } H(p) \quad (4.2)$$

subject to: some p_i 's are equal

$$p \in Q.$$

Hence, the algorithm established in Section 3 does not provide a solution to this maximization problem.

However, as pointed out by S. Fahrenholtz, if one proceeds to use the algorithm and ignores the equality constraint, this would provide a solution to the unrestricted version of the programming problem stated in (4.2) above, and hence an upper bound for its optimal objective function.

4.3 Extended F-event trees

We now introduce the notion of an extended F-event tree. Suppose the components of a system can assume more than two states. Then, for each component utilization, we have a vector $p_k = (p_{k1}, p_{k2}, \dots, p_{km})$ of (conditional) component utilization outcome probabilities. Here, m_k is the number of states that the component at node k can assume, p_{kj} is the (conditional) utilization outcome probability that state j occurs at node k and $\sum_{j=1}^{m_k} p_{kj} = 1$ for all k. The event tree for such a system will be referred to as an extended F-event tree if each p_{ki} appears exactly once.

If E is an event of an extended F-event tree, then the probability of event E can be

expressed as:

$$H(p) = E'(k) + \pi(k) \sum_{j \in I} p_{kj} \pi(s_{kj})$$

where $E'(k)$ and $\pi(k)$ are the quantities defined in (2.5), $\pi(s_{kj})$ is the product of all (conditional) component utilization outcome probabilities of nodes that are involved in event E and that follow p_{kj} in the tree, and I is the index set of all states of node k that occur in event E.

The following theorem is the analogue of Theorems 1 and 2 as applied to extended F-event trees and may be established in a manner similar to the proofs of the later two theorems.

Theorems 3.: Let Q be the cartesian product of regions Q_k for which $p_k \in Q_k$, $k = 1, 2, \dots, n$. Let H be the probability of an event E of an extended F-event tree. Then,

$$\begin{aligned} \max_Q H(p) = & \max_{Q-Q_k} \max_{Q_k} E'(k) \\ & + \pi(k) \sum_{j \in I} p_{kj} \pi^\circ(s_{kj}) \end{aligned}$$

where $\pi^\circ(s_{kj})$ is the value of $\pi(s_{kj})$ that maximizes H.

Now consider the conditional maximization over Q_k . Clearly, the value of p_k that maximizes H can be obtained independently of the values of the arguments that appear in $E'(k)$ and $\pi(k)$. Hence, the solution to this conditional maximization problem is provided by the solution to the programming problem:

$$\max \sum_{j \in I} \pi^\circ(s_{kj}) p_{kj}$$

$$\text{subject to: } \sum_{j=1}^{m_k} p_{kj} = 1$$

$$p_{kj} \in Q_k, \quad j=1, 2, \dots, m_k.$$

5. References

- Barlow, R. E. and H. E. Lambert. 1975. "Introduction to Fault Tree Analysis". in Reliability and Fault Tree Analysis: Theoretical and Applied Aspects of System Reliability and Safety Assessment. IAM, Pennsylvania.
- ven, S. 1973. Algorithmic Combinatorics, The MacMillan Company, New York.
- Guerrero, M.F. and H.T. David, 1985. Optimal Ordering for Buehler Bounds, Philippine Statistician.
- Horowitz, E. and S. Sahni. 1978. Fundamentals of Computer Algorithms Computer Science Press, Maryland.
- Pavlov, I.V. 1980. Confidence bounds for convex functions of many unknown parameters. Theory of Probability and its Applications, 25: 388-393.
- Wong S. 1984. Reliability Analysis for the emergency power system of a pressurized water reactor facility during a loss of offsite power transient. Ph.d. Thesis.